COLLAPSE BEHAVIOUR OF PLATES UNDER THRUST

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Abstract

In line with the current research on the ultimate strength assessment of ship hull girders, careful understanding of the collapse behaviour of unstiffened plate elements within the ship hull girders is an important and necessary task. In this paper, the collapse behaviour of unstiffened plates under longitudinal and transverse thrust is studied by elastoplastic large deflection FEM and the characteristics of the collapse modes are highlighted.

Keywords: Plate- Thrust- Buckling- Ultimate Strength

Introduction

Ship is basically a thin-walled box-girder structure composed of a number of plates and stiffened plates, Fig. 1. Stiffened plate in ships are typically so designed that local panel buckling takes place first, followed by overall buckling of the stiffened plate. Therefore, an accurate model of the plate panel is required to predict its collapse and influence on the behaviour of the stiffened plate as a whole. Such a model is particularly important for the load case of thrust or in-plane compression. Longitudinal bending of the hull girder subjects either the deck or the bottom of a ship to longitudinal thrust. Transverse bending of the bottom structure due to water pressure and cargo weight causes transverse thrust in bottom panels. Plate panels affected by both longitudinal and transverse bending are subjected to biaxial thrust, Fig. 2.

There have been many research works on the collapse behaviour of plate panels, both experimental, e.g. [1], and numerical, e.g. [2-4]. However, in this study, elastoplastic large deflection analyses have been carried out with FEA specifically to clarify the collapse modes of plate panels under thrust.

In what follows, general characteristics of the FEA used in this study and material properties are summarized first. Later, the results of FEA analyses under longitudinal/transverse thrust for square and rectangular plate panels are shown with an emphasize on the characteristics of the collapse modes.

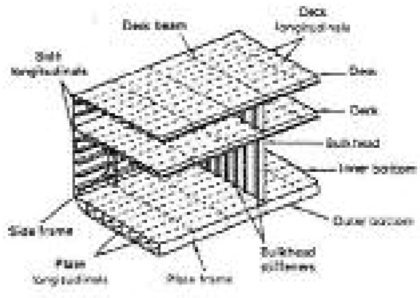

Fig. 1- Ship as a thin-walled box-girder structure

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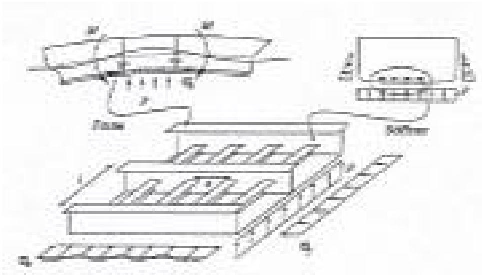


Fig. 2- Plate panels under thrust

FEA Model

All FEA calculations are performed with the in-house code ULSAS. The applicability of ULSAS to the buckling/plastic collapse analysis of steel-plated structures has been examined and confirmed through benchmark calculations at the technical committee of International Ship and Offshore Structures Congress (ISSC), [5-6]. An isoparametric shell element with four nodal points [7] is used in ULSAS. Such element is characterised as bi-linear degenerated element with reduced integration. Both material and geometrical nonlinearities are considered. The analysis using such element has the following characteristics:

(1) The virtual work equation is expressed in terms of updated Green's strain components and the updated Kirchhoff's stress components to derive element stiffness matrix.

(2) The calculated Kirchhoff's stress increments are transformed into the Jaumann's stress increments at each incremental step [8]. In this transformation, the normal strain increment in the thickness direction is determined so that the plane stress condition is maintained. This stress transformation enables to perform ultra large deflection analysis.

(3) The plastic flow theory is applied considering the Mises's yield condition as a plastic potential.

(4) The material is assumed to follow the combined hardening law of kinematics and isotropic hardenings. In the present analysis, however, the stress-strain relationship is assumed to follow that of an elastic-perfectly plastic material.

(5) The stiffness proposed by Kanok-Nukulchai [9] is included in the element stiffness matrix to eliminate singularity regarding the in-plane rigid body rotation.

(6) The stiffness proposed by Flanagon and Belytshko [10] is also included in the element stiffness matrix to prevent hourglass mode instability.

(7) The Crisfield method is employed by the FE code to follow the response in the post-buckling region.

Thrust (in-plane compression) is applied by prescribed uniform in-plane displacement. The material properties used in all analyses within this paper are as follows:

Young's Modulus : E = 205.8 GPa
Poisson's Ratio: $\nu = 0.3$
Yield Strength:
σ_Y = 313.6 MPa
Hardening Rate: $H' = 0$

Geometric models and boundary conditions for plate panels are described in following sections.
For calculations under longitudinal thrust, 10x10 elements per buckling half wave are used. In the case of transverse thrust, the number of elements is: 10 in **y-**direction and the next even number to $a/b*10$ (a : length, b : breadth of plate) in x-direction. Load is applied by forced displacements, and to simulate the continuity of the plating, all edges are kept straight. For square plates, initial deflection are given by one half wave. For rectangular plates, initial deflection in hungry-horse mode is applied.

Plate Panels

The fundamental collapse behaviour of plate panels under thrust is investigated by FEA in this paper. A simply supported plate, Fig. 3, has been used in analyses. This model corresponds to the plating between longitudinal and transverse members.
It is obvious that stiffeners have important effect on the buckling/plastic collapse behaviour of plates, owing to their bending as well as torsional rigidities. When the plate part of stiffened panels is being modeled under the action of in-plane compression, simply supported plate model is sufficient in accuracy having considered straight-edge conditions along plate edges [11]. Keeping edges straight is a necessity to satisfy the condition of continuity of the plating in actual plated structures. Besides, when the plate behaviour under the action of combined in-plane compression and lateral pressure is going to be studied, the continuous plate models are to be applied in FEA [11].
For most ship panels the proportions are such designed that the local plate buckling precedes both local stiffener tripping or overall buckling of stiffened plate. That is why this work focuses on the plates. On the other hand, such analyses performed in this work give a good indication of the likely modes of failure and also provide a foundation for the most complex question of collapse behaviour of stiffened panels.

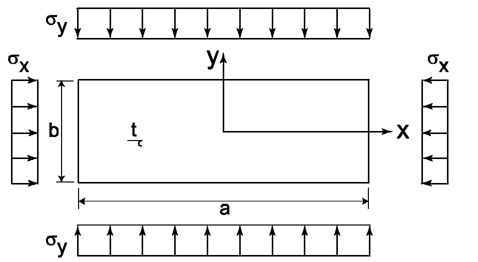


Fig. 3- Simply supported plate under thrust

Square Plate under Uniaxial Thrust

As a most fundamental case, a square plate under uniaxial thrust in x-direction is considered. The plate has been given an initial deflection with a magnitude of 10% of plate thickness in its buckling mode. This is a typical value and is smaller than the average value of initial deflection of ship plating ($0.05\beta^2 t$) proposed by Varghese[12]. Average stress-average strain relationship of the plate has been shown in Fig. 4, in which the values of average stress (σ_x) and average strain (ε_x) have been nondimensionalised respectively by material yield stress (σ_Y) and yield strain ($\varepsilon_Y = \sigma_Y / E$). As shown in Fig. 4, after buckling point (the point corresponding to the sudden change in the slope of the stress-strain curve or first loss of stiffness) and initial yielding, the plate reaches its ultimate strength. Afterwards, the load-carrying capacity decreases with increasing applied average strain. Fig. 5 shows shape of deflection and spread of yielding at ultimate strength and in the post-ultimate range.

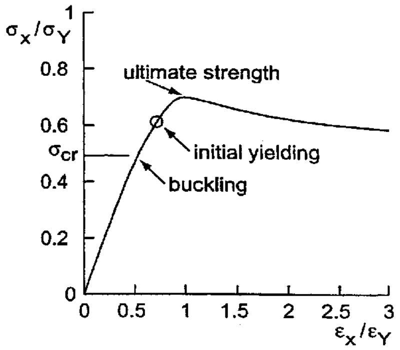
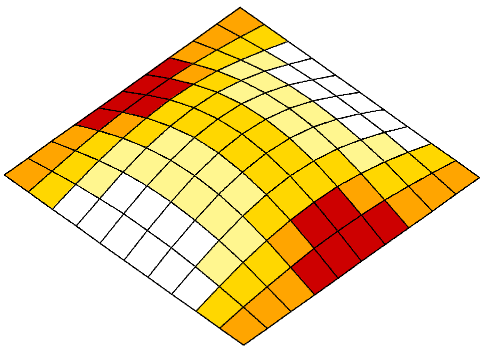
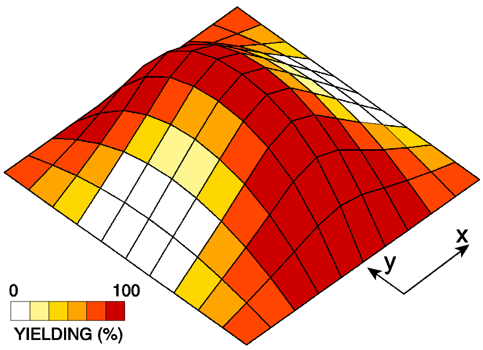


Fig. 4- Average stress-average strain relationship of a square plate (1000x1000x15.5 mm) under thrust



(a) ultimate strength, $\varepsilon_x / \varepsilon_Y$=0.98



(b) post-ultimate, $\varepsilon_x / \varepsilon_Y$=3.0

Fig. 5- Collapse mode of square plate (1000x1000x15.5 mm) under thrust

At ultimate strength, the deflected shape is still very similar to the sinusoidal buckling mode. However, the collapse takes place due to the spread of yielding along the unloaded edges. The in plane strain associated with this yielding straightens the sinusoidal shape in the direction of applied load. Beyond ultimate strength, deflection in a roof mode is clearly observed.

The deflected shape of a square plate can be expressed over the whole range of loading as

$$w = \sum_i \sum_j A_{ij} \sin\frac{i\pi x}{a} \sin\frac{j\pi y}{b} \qquad (1)$$

The coefficients of this Fourier series change with the state of loading. For the example considered here, the higher-order coefficients divided by A_{11} (the coefficient corresponding to the buckling mode) are shown in Fig. 6 depending on the average applied strain (nondimensionalized by yield strain). All other coefficients are either zero or very small. During the initial loading stages, all higher-order terms are negligible, the plate has a sinusoidal shape. Beyond ultimate strength, A_{31} causes the straightening in x-direction, leading to the roof mode. The increase in A_{13} tends to produce a plateau in y-direction, but in general the deflected shape stays sinusoidal in y-direction. A_{33} can be seen as coupling term of these effects.

The change from a sinusoidal shape to a roof mode beyond ultimate strength is a characteristic phenomenon for a wide range of thickness. Only very thick plates practically remain in a sinusoidal mode, and extremely thin plates, which are not considered here, collapse due to secondary buckling in a higher eigen mode. For all other plates, it can be said: the thinner the plate is, the more emphasized the roof mode becomes and the earlier it starts to develop.

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Rectangular Plate under Longitudinal Thrust

Rectangular plates in actual ships have a complex initial deflection in the so-called hungry-horse mode [13], Fig. 7. This initial deflection can be expressed as

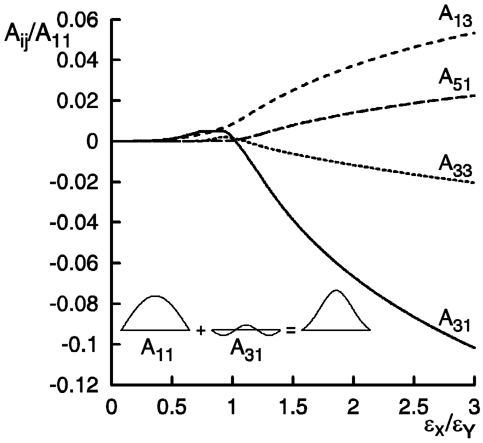


Fig. 6- Fourier coefficients of deflection for square plate (1000x1000x15.5 mm) depending on average strain

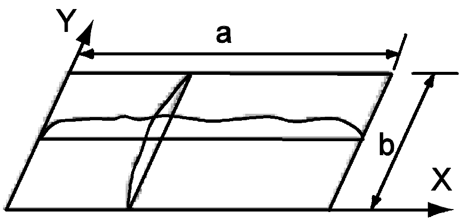


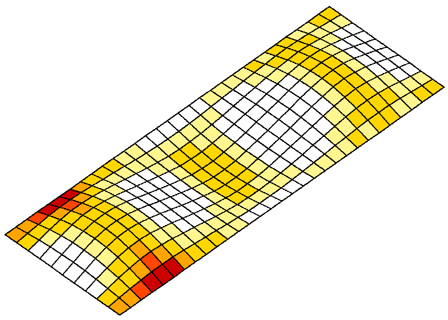
Fig. 7- Hungry-horse mode initial deflection [13]

$$w_0 = \sum_i A_{0i} \sin\frac{i\pi x}{a} \sin\frac{\pi y}{b} \qquad (2)$$

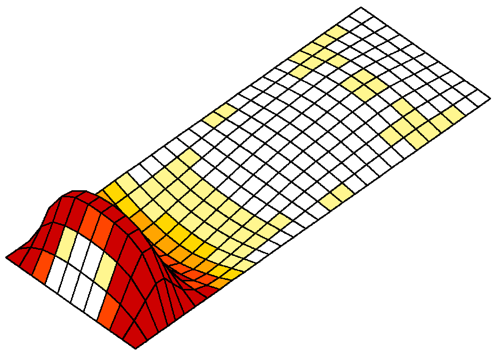
Where the values of the coefficients in mm, are given in Table 1 for the plate panel considered here [13]. This initial deflection is directly applied to the FEA model.

A rectangular plate under longitudinal thrust buckles into several half waves. Figure 8 shows a rectangular plate (2400x835x15 mm) which has undergone elastic buckling in three half waves. The sinusoidal buckling mode is still present at ultimate strength, but due to the nonuniform initial deflection of hungry-horse mode, the deflection is not the same in different half waves. These differences in the deflection correspond to different extents of yielding. One half wave reaches its ultimate strength first. This half wave collapses whereas the rest of the plate unloads; the plastic deformation is localized beyond ultimate strength.

The collapsing half wave of the rectangular plate basically shows the same behaviour as an equivalent square plate. The deflection mode changes from a sinusoidal shape to a roof mode.



(a) ultimate strength, $\varepsilon_x / \varepsilon_Y$ =0.89



(b) post-ultimate, $\varepsilon_x / \varepsilon_Y$ =3.0

Fig. 8- Collapse mode of rectangular plate (2400x835x15 mm) under longitudinal thrust (hungry -horse mode initial deflection)

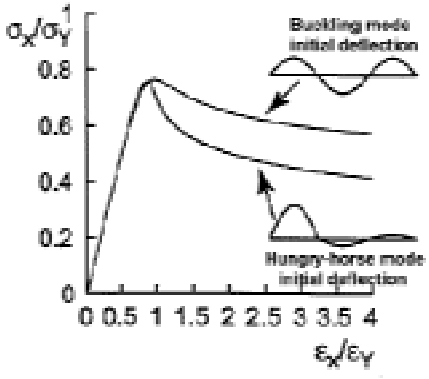


Fig. 9- Average stress-average strain relationship of a rectangular plate (2400x835x15 mm) under longitudinal thrust

As a result of localization, the drop of load-carrying capacity in the average stress-average strain curve is rapid, Fig. 9. For comparison, the stress-strain path of the same plate, but starting from a uniform sinusoidal initial deflection in its buckling mode is also shown. The initial deflection of the plate in such a case is expressed as

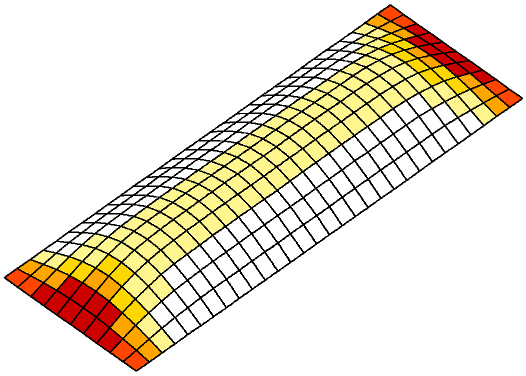
$$w_0 = A_{03} \sin\frac{3\pi x}{a} \sin\frac{\pi y}{b} \quad (3)$$

In which the value of A_{03} is taken from Table 1, and the values of a and b are respectively equal to 2400 mm and 835 mm.

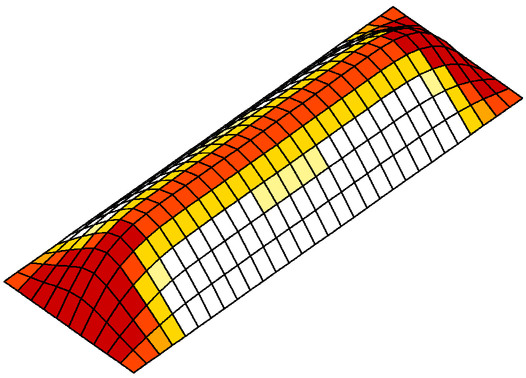
Rectangular Plate under Transverse Thrust

Under transverse thrust, a rectangular plate buckles in one big half wave, and the transverse buckling strength is lower than the longitudinal one. The same rectangular plate as under longitudinal thrust is considered here. Unlike the case of longitudinal thrust, the sinusoidal form has already changed at ultimate strength, Fig. 10. The middle part of the plate has a cylindrical shape, sinusoidal in transverse direction, but longitudinally constant. The end parts collapse as half of a square plate each. Beyond ultimate strength, the shape is straightened in loading direction, also forming a kind of roof mode.

Nevertheless, the most important feature of the collapse under transverse thrust is that the cylindrical middle part is formed before reaching ultimate strength. Figure 11 shows a typical average stress-average strain relationship of the plates under transverse thrust. Only plates, which are so thick that they undergo plastic buckling even under transverse thrust, start forming the cylindrical shape beyond ultimate strength.



(a) ultimate strength, $\varepsilon_y / \varepsilon_Y$ =1.05



(b) post-ultimate, $\varepsilon_y / \varepsilon_Y$ =3.0

Fig. 10- Collapse mode of rectangular plate (2400x835x15 mm) under transverse thrust

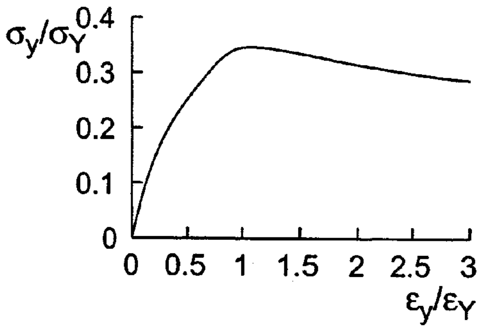


Fig. 11- Average stress-average strain relationship of a rectangular plate (2400x835x15 mm) under transverse thrust

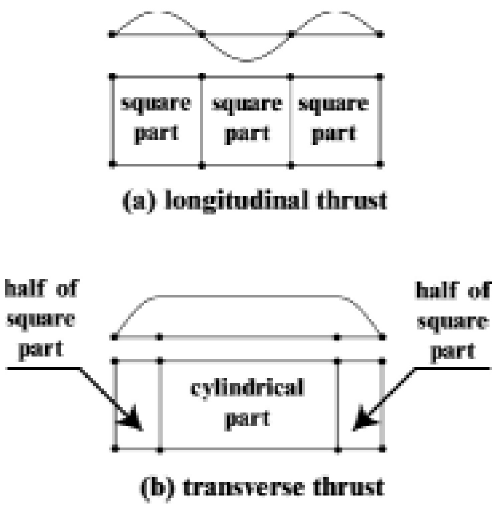


Fig. 12- Idealised plate models

Idealization of the Plate Behaviour

The model shown in Figure 12 may be regarded as an idealized model to simulate rectangular plate behaviours under the action of longitudinal or transverse thrust. The rectangular plate under longitudinal thrust is equally divided by as many square (or nearly square) elements as the number of buckling half waves, n . In case of transverse loading, the model consists of two end parts and a middle part. Both end parts behave as a square plate, while the middle part is collapsing in a cylindrical manner.

Conclusions

The characteristics of the collapse behaviour of rectangular plates under longitudinal and transverse thrust have been pointed out with elastoplastic large deflection FEM analyses. The main findings can be summarized as

1. Square plates under uniaxial thrust; change from their sinusoidal buckling mode to a roof mode beyond ultimate strength.
2. Rectangular plates under longitudinal thrust show localized plastic deformation when collapsing. The collapsing half wave basically behaves like a square plate beyond ultimate strength.
3. Under transverse thrust, the collapse mode of most rectangular plates is significantly different from their buckling mode. Even before ultimate strength, a cylindrical shape in the middle of the plate develops.
4. An idealized model to simulate rectangular plate behaviours under the action of longitudinal or transverse thrust was proposed. Such a model is to be used in the formulation of plate elements of the so-called Idealised Structural Unit Method (ISUM) [14] developed for the analysis of large plated structures.

Table 1: Coefficients of hungry-horse mode for a plate panel of 2400x835x15 mm [mm]

A_{01}	A_{02}	A_{03}	A_{04}	A_{05}	A_{06}	A_{07}	A_{08}	A_{09}	$A_{0\,10}$	$A_{0\,11}$
3.9553	0.2182	1.2040	0.0949	0.3592	0.0248	-0.0228	0.0803	-0.1374	-0.0163	-0.0153

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