DEVELOPING A FORMULA TO REPRESENT THE SECOND-ORDER WAVE EFFECTS ON MOORED FLOATING STRUCTURES

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Abstract

The effects of wave direction, frequency and the waterline shape of floating structures on the wave mean drift force formula have been considered separately by several authors but there isn't a general formula to take into account all of these effects. In this regard, Faltinsen's wave drift force formula has been modified by adding finite draft coefficient. The results obtained from this formula which is dependent on wave frequencies, has been compared with Helvacioglu's experiments favorably in high wave frequencies. Moreover, the influence of the current on the wave mean drift force has been taken into account by considering the current coefficient derived from the ship added resistance formula. In addition, the formula for the calculation of the wave drift damping has been extended to cover high wave frequencies as well as low wave frequencies. The results compared with asymptotic formula showed good agreement in the high frequency band.

Keywords: Hydrodynamics, Second-order wave effects, Floating structures

1. Introduction

In determining the total mooring loads and motions of moored floating structures such as FPSOs (Floating Production, Storage and Offloading systems) and SPMs (Single Point Mooring systems), in addition to the first order wave effects, the second-order effects are also important. The second-order wave effects yield additional forces and responses which are proportional to the square of wave amplitudes. The presence of the second-order effects can simply be illustrated from Bernoulli's equation by considering its quadratic velocity term [1]. Helvacioglu[2] has done some experiments in order to obtain drift force coefficients. The second-order wave forces can be derived from pressure integration (near field) approach [3]. More details about the derivation can be found in Pinkster [4] , Standing et al. [5], Tong [6] and Ogilive [7]. The solution of the second-order effects leads to mean drift and slow varying loads. This is shown here by employing the velocity potential described by Pinkster [4]. Havelock [8] presented an empirical formula for calculating the wave drift force acting on a fixed cylinder with vertical walls which has been extended by Besho [9] to cover arbitrary waterline shape bodies. Then Kwon [10] extended the Besho's formula by including three coefficients to take into account the effects of speed, finite draft and scattering. Faltinsen [1] presented a general frequency independent formula for calculating the mean drift loads due to arbitrary waves on arbitrary waterline shaped bodies. It can be noted that the above authors have tried to include partially the effects of wave direction, wave frequency, current and arbitrary waterline shape bodies on their mean

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drift force formulas. But, there isn't one general formula that can take into account the aforementioned effects. Therefore, in order to take into account all of the above effects, Faltinsen's wave drift force formula is modified and redeveloped by adding the finite draft coefficient. The results obtained from the redeveloped formula are compared with Helvacioglu's experiments [2]. Moreover, the influence of the current on the wave mean drift force is calculated by using the current coefficient derived from the ship added resistance theory.

2.SECOND-ORDER WAVE FORCES AND MOMENTS

The theory of second-order wave forces and moments has been developed on the assumption that the fluid surrounding the body is inviscid, irrotational, homogenous and incompressible. Therefore, the fluid motions can be described by a velocity potential, as [4]:

$$\phi = \sum_{i=1}^{n} \varepsilon_i \phi_i \qquad (1)$$

Where
ε_i is a small parameter (perturbation)
ϕ_i is the ith order velocity potential so for example, ϕ_2 denotes the second order velocity potential
Now by considering a fixed coordinate system the pressure in a point on the hull of a floating structure (e.g. FPSO) can be determined by writing down the Bernoulli's equation as:

$$p = P_0 - \rho g z - \rho \frac{\partial \phi}{\partial t} - \frac{1}{2}\rho |\nabla \phi|^2 \qquad (2)$$

Where
P_0 is the static pressure (atmospheric pressure),
∇ is the gradient.

The quadratic term of Eq.(2) can be extended as:

$$-\frac{1}{2}\rho |\nabla \Phi|^2 = -\frac{1}{2}\rho \left| V_1^2 + V_2^2 + V_3^2 \right| \qquad (3)$$

Where
V_1,are velocity terms related to the directions of axes of coordinate system.
Extending the first velocity term of Eq. (3) for two wave components with different wave amplitudes A_1 and A_2, and different circular frequencies ω_1 and ω_2 propagating in an idealized sea state leads to [1]:

$$\frac{-\rho}{2} V_1^2 = \qquad (4)$$
$$\frac{-\rho}{2}\left(\frac{A_1^2}{2} + \frac{A_2^2}{2} + \frac{A_1^2}{2}\cos\left(2\omega_1 t + 2\varepsilon_1\right)\right)$$
$$\frac{-\rho}{2}\frac{A_2^2}{2}\cos\left(2\omega_2 t + 2\varepsilon_2\right)$$
$$\frac{-\rho}{2} A_1 A_2 \cos\left((\omega_1 - \omega_2)t + \varepsilon_1 - \varepsilon_2\right)$$
$$\frac{-\rho}{2} A_1 A_2 \cos\left((\omega_1 + \omega_2)t + \varepsilon_1 + \varepsilon_2\right)$$

Eq. (4) shows that second order effects are generally those effects which are either linear with the wave amplitude or proportional to the square of wave amplitude. The above equation can be described by the following terms

$-\frac{\rho}{2}\left(\frac{A_1^2}{2} + \frac{A_2^2}{2} \right)$:

This term represents steady pressure.

$$\frac{\rho}{2}\frac{A_1^2}{2}\cos\left(2\omega_1 t + 2\varepsilon_1\right) +$$
$$\frac{\rho}{2}\frac{A_2}{2}\cos\left(2\omega_2 t + 2\varepsilon_2\right) +$$
$$\frac{\rho}{2} A_1 A_2 \cos\left([\ \omega_1 + \omega_2\ ]t + \varepsilon_1 + \varepsilon_2\right)$$

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This term shows the non-linear effects which can excite a structure with frequencies higher than the dominant frequency components in a wave spectrum.

$$-\frac{\rho}{2}\left(A_1 A_2 \cos\left[(\omega_1-\omega_2)t+\varepsilon_1-\varepsilon_2\right]\right):$$

This term shows that the non-linear effects can also oscillate a body with difference frequency such as $(\omega_1-\omega_2)$.

In other words, the presence of constant term, pressure term oscillating with different frequencies and the terms with higher frequencies than dominant frequency components in a wave spectrum, clearly represent the effects of the second-order wave loads and moments. Mean (Drift) and also slowly varying loads and moments are the results of the second order effects. It should be also noted that the high wave frequency excitation loads and moments are other results of second order effects which may be important for calculating TLPs' motion particularly in heave, pitch and roll [1].

3. MEAN DRIFT LOADS

Havelock [8], presented an empirical formula for calculating the wave drift force acting on a fixed cylinder with vertical sides. This formula which was derived from the quadratic term of the velocity potential is as follows:

$$F_x = \tfrac{1}{2}\rho g\zeta_a^2 \int_{-b}^{b}\sin^2\gamma_H dy = \tfrac{1}{2}\rho g\zeta_a^2 B\overline{\sin^2\gamma_H} \qquad (5)$$

Where

γ_H : is the tangential slope of the water plane curve with respect to longitudinal centre line.

B : is the diameter of the cylinder

ζ_a : is the wave amplitude

Also, the bar denotes the mean value.

After that Besho [9] extended the above formula and obtained a generalized drift force formula for an arbitrary waterline shape bodies subjected to very short waves. The Besho's formula can be written as:

$$F_x = \tfrac{1}{2}\rho g\zeta_a^2 \int_{Lr}\left(\frac{\partial y}{\partial l}\right)^3 dl \qquad (6)$$

Where

$\frac{\partial y}{\partial l}$: is the same as $\sin\gamma_H$ in the Havelock's formula

L_r: is the exposed water line part of a floating structure as shown in Fig. 1.

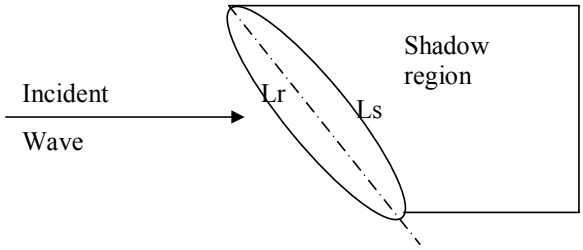


Fig. 1- The exposed and the shadow part of a floating structure subjected to an arbitrary wave

Pinkster and Oortmerssen [11], presented the direct pressure integration method to obtain the mean wave forces and moments. By considering a two-dimensional piercing body subjected to a very small wave length it can be assumed that the hydrodynamic behavior of the sea wave around the body is the same as the hydrodynamic pattern around a vertical plane wall (Fig. 2).

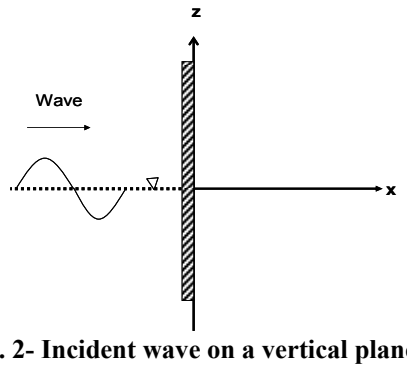


Fig. 2- Incident wave on a vertical plane wall

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The linear solution according to Faltinsen [1] can be written as

$$\phi_1 = \frac{2g\zeta_a}{\omega} e^{kz} \cos\omega t \cos kx \qquad (7)$$

Now it could be possible to write the Bernoulli's equation up to second order terms in wave amplitude as:

$$p = -\rho g z - \rho \frac{\partial \phi_1}{\partial t} - \frac{\rho}{2}\left(\left(\frac{\partial \phi_1}{\partial y}\right)^2 + \left(\frac{\partial \phi_1}{\partial z}\right)^2\right)$$

The first term can be calculated at the wall as:

$$\overline{-2\rho g \int_0^{\zeta_a} z dz} = -2\rho g \left[\tfrac{1}{2} z^2\right]_0^{\zeta_a} = -\rho g \zeta_a^{\,2}$$

Also, the second term can be simplified as:

$$\overline{-\rho\left[\frac{\partial \phi_1}{\partial t}\right]_{z=0} \zeta_a} = -\rho\left[-2g\zeta_a\right]\zeta_a = 2\rho g \zeta_a^{\,2}$$

And finally the third term can be written as:

$$\overline{-\frac{\rho}{2}\int_{-\infty}^{0}\left\{\left(\frac{\partial \phi_1}{\partial y}\right)^2 + \left(\frac{\partial \phi_1}{\partial z}\right)^2\right\} dz} =$$

$$-\frac{\rho}{2}\int_{-\infty}^{0}\left\{\left(\frac{1}{2}\frac{4g^2\zeta_a^{\,2}}{\omega^2} k^2 e^{2kz}\right)\underbrace{\left[\sin^2 ky + \cos^{2ky}\right]}_{=1}\right\} dz$$

$$= -\frac{\rho}{2} g \zeta_a^{\,2} \int_{-\infty}^{0} 2k e^{2kz} dz = -\frac{1}{2}\rho g \zeta_a^{\,2}$$

Adding up the above three terms will lead to $(\rho g / 2)\zeta_a^2$.

Faltinsen [1] also showed that the result of the above equation: $(\rho g / 2)\zeta_a^2$ which is the correct asymptotic value for small wavelengths according to Marou's formula [12]. Faltinsen [1] has also showed that the above result can be validated for other structures where the assumption of vertical sides at water plane area can be made. So, for a vessel shape structure subjecting to an arbitrary wave (

Fig. 3), the mean drift forces and moments can be calculated through the following formula:

$$F_i = \frac{\rho g \zeta_a^{\,2}}{2} \int_{L_1} \sin^2(\theta + \gamma) n_i dl \qquad (8)$$

Where;

F_1: is the wave surge drift force component

F_2: is the wave sway drift force component

F_6: is the wave yaw drift moment component

$n_1 = \sin\theta$

$n_2 = \cos\theta$

$n_6 = x\cos\theta - y\sin\theta$

L_1: is the integration domain, the non shadow part of the water plane curve.

γ: is the wave propagation angle with respect to x axis

θ: is tangential angle of water plane curve

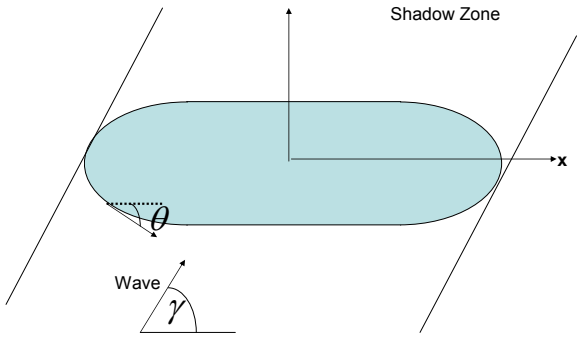


Fig. 3- Water plane area and the shadow zone of a vessel shape structure subjected to an arbitrary wave

Eq.(8) is similar to the Besho's equation except that, the yaw drift moment has been included in the Faltinsen's formula. Also the role of incident wave direction

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on the drift force can be seen more clearly in the above formula. Kwon [10], proposed a new formula by extending the Besho's equation to calculate wave drift forces. He added three coefficients which are as follows:

C_v: Correction factor for advance speed

C_s: Scattering coefficient

C_T: Correction factor for finite draft

which can be defined as:

$C_T = 1 - \exp(-2kD)$

where k is the wave number and D is the draft of the vessel

For a stationary barge shape structure with a beam of B subjecting to the head sea waves the surge drift force according to Eq.(8) can be written as:

$$F_1 = \frac{\rho g \zeta_a^2}{2} \int_{L_1} \sin^2(\theta + 0) n_1 dl$$

Where $\theta = 90$, $n_1 = 1$, $dl = dy$ and L_1 is the integration domain which for this case is the floating structure's beam. Therefore the above formula would become:

$$F_1 = \frac{\rho g \zeta_a^2}{2} B$$

Now by adding the Kwon's finite draft coefficient to the formula the surge drift force can be written as:

$$F_1 = C_T \frac{\rho g \zeta_a^2}{2} B \qquad (9)$$

Now by defining surge drift force coefficient as follows:

$$R_1(\omega) = \frac{F_1}{\frac{1}{2}\rho g \zeta_a^2 B} \qquad (10)$$

and substituting F_1 from Eq.(9) the surge drift force coefficient can be simplified as Eq.(11). This means that for a stationary rectangular barge shape vessel subjected to a head sea waves the square of the surge drift force coefficient is equal to the finite draft coefficient.

$$R_1(\omega) = C_T \qquad (11)$$

The variation of the finite draft coefficient "C_T" with wave frequency for different draft values are shown in Fig. 4. It is clear from the figure that an increase in structure's draft would lead to an increase in the finite draft coefficient. This effect can be seen more in average wave frequencies (i.e. 0.4-0.8 rad/sec.). The surge drift force coefficient, has been compared with those obtained by Kwon[10], Fujii and Takahashi [13], and Helvacioglu [2] as shown in Error! Reference source not found.. The comparisons showed that the result of Eq.(9) is the same as Kwon's results which have a good correlation with Helvacioglu's experiments for high wave frequencies.

3.1.THE EFFECT OF CURRENT ON WAVE DRIFT LOADS

In order to take into account the influence of current on wave drift loads, the ship added resistance formula proposed by Faltinsen, Minsaas et. al. [14] is employed. Therefore, the current coefficient can be defined as:

$$C_{cu,i} = 1 + \frac{2\omega U_i}{g} \qquad (12)$$

where

$U_1 = U \cos\beta$

$U_2 = U \sin\beta = U_6$

U: is the current speed

β: is the current angle in respect to the x axis.

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Eq.(12) can be employed for bluff bodies when Froude number is equal or less than 0.2 ($F_n \leq 0.2$).

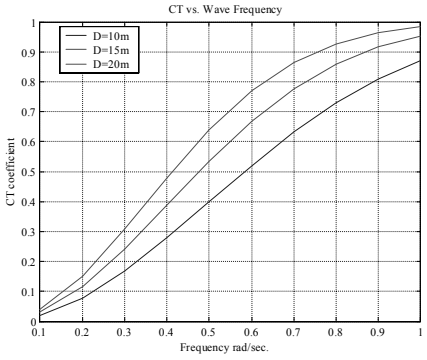


Fig. 4-The variation of C_T against wave frequency for different drafts

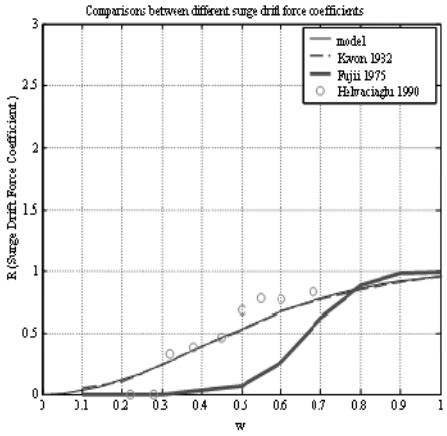


Fig. 5-Different surge drift force coefficients

Eq. (12) can be also written as:

$$C_{cu,i} = 1 + 2\omega F_{ni}\sqrt{\frac{D_i}{g}} \qquad (13)$$

where

$$F_{ni} = \frac{U_i}{\sqrt{gD_i}}$$

D_i is the dimension of the obstacle in the direction of current and it is equal to the length of the vessel in the surge, and beam of the vessel in the sway and yaw modes.

Eq.(13) can clearly show how the current speed, wave frequency and the dimension of structure can influence the current coefficient. However, in reality the current coefficient is not affected by the dimension of a structure and this can be seen by looking back to the original equation which is Eq.(12). So, the magnitude of current coefficient is only dependent on the wave frequency and current speed. For example for an offshore structure with 100m diameter subjecting to waves with circular frequencies between 0.1 and 1 rad/sec and current with the speed of 1 m/sec, the current coefficient will vary between 1.02 and 1.20. The same results can be achieved for an offshore structure with 20m diameter.

As an example, the derived current coefficient has been applied to a 200,000 tone dead weight floating offshore structure (i.e FPSO). The result showed that existing a 2 m/s current can increase the drift loads up to 40 percent in high wave frequencies.

3.2.DERIVING STEADY DRIFT FORCES AND MOMENTS FOR AN FPSO

The steady drift forces and moments for an FPSO subjected to arbitrary waves and current by considering finite draft coefficient "C_T" and current coefficient "C_{cu}" mentioned in Eq.(11) and Eq.(12) respectively, can be extended as follows:

$$F_i = C_T C_{cu,i}\frac{\rho g \zeta_a^2}{2}\int_{L_1}\sin^2(\theta+\gamma)n_i dl \qquad (14)$$

So the steady surge and sway drift forces and yaw drift moment can be written as:

Surge:

$$F_1 = C_T\left(1+\frac{2\omega U\cos\beta}{g}\right)\frac{\rho g\zeta_a^2}{2}\int_{L_1}\sin^2(\theta+\gamma)\sin\theta dl \qquad (15)$$

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Sway:

$$F_2 = C_T\left(1+\frac{2\omega U\sin\beta}{g}\right)\frac{\rho g\zeta_a^2}{2}\int_{L_1}\sin^2(\theta+\gamma)\cos\theta\,dl \qquad (16)$$

Yaw:

$$F_6 = C_T\left(1+\frac{2\omega U\sin\beta}{g}\right)\frac{\rho g\zeta_a^2}{2}\int_{L_1}\sin^2(\theta+\gamma)(x\cos\theta - y\sin\theta)\,dl \qquad (17)$$

Considering an FPSO with a general water plane area shown in Fig. **6** and assuming that the most left and right end curvature parts can be replaced by a half-circle with a diameter equal to the beam of the middle rectangular section, Eq.(15) ,Eq.(16) and Eq.(17) can be more developed as follows:

Surge Drift Force

$$F_1 = C_T\left(1+\frac{2\omega U\cos\beta}{g}\right)\frac{\rho g\zeta_a^2}{2}\int_{L_1}\sin^2(\theta+\gamma)\sin\theta\,dl$$

$$F_1 = C_T\left(1+\frac{2\omega U\cos\beta}{g}\right)\frac{\rho g\zeta_a^2}{2}\left(\underbrace{\int_I \sin^2(\theta+\gamma)\sin\theta\,dl_I}_{part1} + \underbrace{\int_{II}\sin^2(\theta+\gamma)\sin\theta\,dl_{II}}_{part2} + \underbrace{\int_{III}\sin(\theta+\gamma)\sin\theta\,dl_{III}}_{part3}\right)$$

Part 1:

$$dl_I = r\,d\theta$$

$$\int_0^{\pi-\gamma}\sin^2(\theta+\gamma)\sin\theta\,r\,d\theta =$$

$$r\left[\frac{2}{3}\cos\gamma - \frac{1}{3}\sin^2\gamma + \frac{2}{3}\right]$$

where $r = \frac{B}{2}$

Part 2:

$$dl_{II} = dx;\ \theta = 0$$

$$\int_{II}\sin^2(0+\beta)\sin(0)\,dx = 0$$

Part 3:

$$dl_{III} = r\,d\theta$$

$$\int_{-\beta}^{0}\sin^2(\theta+\gamma)\sin\theta\,r\,d\theta =$$

$$r\left[\frac{1}{3}\sin^2\beta - \frac{2}{3} + \frac{2}{3}\cos\beta\right]$$

Therefore by summing up all of the above parts, the surge drift force will become:

$$F_1 = C_T\left(1+\frac{2\omega U\cos\beta}{g}\right)\frac{\rho g\zeta_a^2}{2}\left(\underbrace{r\left[\frac{2}{3}\cos\gamma - \frac{1}{3}\sin^2\gamma + \frac{2}{3}\right] + 0 + r\left[\frac{1}{3}\sin^2\beta - \frac{2}{3} + \frac{2}{3}\cos\beta\right]}_{\frac{4}{3}r\cos\gamma}\right)$$

$$F_1 = \frac{2}{3}C_T\left(1+\frac{2\omega U\cos\beta}{g}\right)\rho g\zeta_a^2 r\cos\gamma \qquad (18)$$

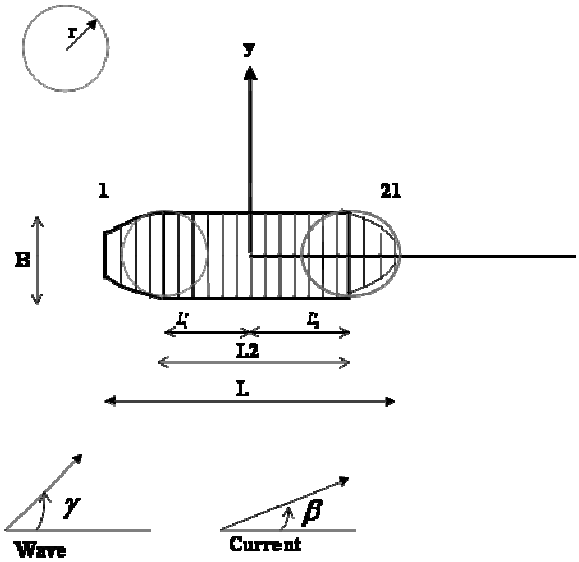


Fig. 6-The water plane area of a 200,000 dtw FPSO subjected to wave and current

Sway Drift Force

$$F_2 = C_T\left(1+\frac{2\omega U\sin\beta}{g}\right)\frac{\rho g\zeta_a^2}{2}\int_{L_1}\sin^2(\theta+\gamma)\cos\theta\,dl$$

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$$F_2 = C_T\left(1+\frac{2\omega U\sin\beta}{g}\right)\frac{\rho g\zeta_a^2}{2}$$
$$\left(\underbrace{\int_I \sin^2(\theta+\gamma)\cos\theta dl_I}_{part\,4} + \underbrace{\int_{II}\sin^2(\theta+\gamma)\cos\theta dl_{II}}_{part\,5} + \underbrace{\int_{III}\sin(\theta+\gamma)\cos\theta dl_{III}}_{part\,6}\right)$$

Part 4:

$$dl_I = rd\theta$$
$$\int_0^{\pi-\gamma}\sin^2(\theta+\gamma)\cos\theta rd\theta =$$
$$r\left[\frac{2}{3}\sin\gamma+\frac{2}{3}\sin\gamma\cos\gamma\right]$$

Part 5:

$$dl_{II} = dx;\ \theta = 0$$
$$\int_{II}\sin^2(0+\beta)\cos(0)dx = L_2\sin\beta|\sin\beta|$$

where L_2 is the length of the middle rectangular part of an FPSO (Fig. 6).

Part 6:

$$dl_{III} = rd\theta$$
$$\int_{-\beta}^{0}\sin^2(\theta+\gamma)\cos\theta rd\theta =$$
$$r\left[-\frac{2}{3}\sin\gamma\cos\gamma+\frac{2}{3}\sin\beta\right]$$

So we can rewrite the sway drift force as:

$$F_2 = C_T\left(1+\frac{2\omega U\sin\beta}{g}\right)\frac{\rho g\zeta_a^2}{2}$$
$$\left(\underbrace{r\left[\frac{2}{3}\sin\beta+\frac{2}{3}\sin\beta\cos\beta\right]+ L_2\sin\beta|\sin\beta|+r\left[-\frac{2}{3}\sin\beta\cos\beta+\frac{2}{3}\sin\beta\right]}_{\frac{4}{3}r\sin\beta+L_2\sin\beta|\sin\beta|}\right)$$

or

$$F_2 = C_T\left(1+\frac{2\omega U\sin\beta}{g}\right)\frac{\rho g\zeta_a^2}{2}$$
$$\left(\frac{4}{3}r\sin\beta+L_2\sin\beta|\sin\beta|\right) \qquad (19)$$

Yaw Drift Moment

$$F_6 = C_T\left(1+\frac{2\omega U\sin\beta}{g}\right)\frac{\rho g\zeta_a^2}{2}$$
$$\int_{L_1}\sin^2(\theta+\gamma)(x\cos\theta-y\sin\theta)dl$$

By extending the integration part of the above formula, it becomes:

$$F_6 = C_T\left(1+\frac{2\omega U\sin\beta}{g}\right)\frac{\rho g\zeta_a^2}{2}$$
$$\left(\underbrace{\int_I \sin^2(\theta+\gamma)(x\cos\theta-y\sin\theta)dl_I}_{part\,7} + \underbrace{\int_{II}\sin^2(\theta+\gamma)(x\cos\theta-y\sin\theta)dl_{II}}_{part\,8}\right)$$
+
$$\left(\underbrace{\int_{III}\sin(\theta+\gamma)(x\cos\theta-y\sin\theta)dl_{III}}_{part\,9}\right)$$

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Part 7:

$$dl_I = rd\theta;\ x = -L_1{}' - r\sin\theta;$$
$$y = -r\cos\theta$$
$$\int_{\pi-\gamma}^{0}\sin^2(\theta+\gamma)\left[\left(-L_1{}' - r\sin\theta\right)\cos\theta-(-r\cos\theta)\sin\theta\right]rd\theta$$
$$= -rL_1{}'\left[-\frac{2}{3}\sin\gamma-\frac{2}{3}\sin\gamma\cos\gamma\right]$$

Part 8:

$$dl_{II} = dx;\ \theta = 0$$
$$\int_{II}\sin^2(0+\beta)(x\cos 0-y\sin 0)dx$$
$$= \frac{L_2{}'^2 - L_1{}'^2}{2}\sin\gamma|\sin\gamma|$$

Part 9:

$$dl_I = rd\theta;\ x = L_2{}' + r\sin\theta;$$
$$y = -r\cos\theta$$
$$\int_0^{-\beta}\sin^2(\theta+\gamma)\left[\left(L_2{}' + r\sin\theta\right)\cos\theta-(-r\cos\theta)\sin\theta\right]rd\theta$$

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$$= rL_2{}'\left[-\frac{2}{3}\sin\gamma+\frac{2}{3}\sin\gamma\cos\gamma\right]$$

$$F_6 = C_T\left(1+\frac{2\omega U\sin\beta}{g}\right)\frac{\rho g\zeta_a^2}{2}$$

$$\left[\begin{array}{l} -rL_1{}'\left(-\frac{2}{3}\sin\gamma-\frac{2}{3}\sin\gamma\cos\gamma\right)+ \\ \frac{L_2{}'^2-L_1{}'^2}{2}\sin\gamma\left|\sin\gamma+rL_2{}'\left(-\frac{2}{3}\sin\gamma+\frac{2}{3}\sin\gamma\cos\gamma\right)\right| \end{array}\right]$$

$$F_6 = C_T\left(1+\frac{2\omega U\sin\beta}{g}\right)\frac{\rho g\zeta_a^2}{2}$$

$$\left[\begin{array}{l} \frac{1}{3}r\sin\gamma\left(L_1{}'-L_2{}'\right)+\left(L_2{}'^2-L_1{}'^2\right)\sin \\ \frac{1}{6}\left(rL_1{}'+rL_2{}'\right)\sin 2\gamma \end{array}\right] \quad (20)$$

It is obvious that all of the derived drift loads in Eq.(18), Eq.(19) and Eq.(20) are linearly dependent on the finite draft coefficient "C_T". So, it can be concluded that an increase in the finite draft coefficient will lead to an increase in drift loads as well.

3.3.DRIFT WAVE LOADS IN IRREGULAR WAVES

The drift wave loads in irregular waves can be determined as follows:

$$\overline{F_i^s} = 2\int_0^\infty s(\omega)\left(\frac{F_i(\omega,\beta,\gamma)}{\zeta_a^2}\right)d\omega \quad (21)$$

$$i = 1,...,6$$

where

$S(\omega)$ is the sea spectrum

$F_i(\omega,\beta,\gamma)$ is the i[th] mean force component in regular waves with the circular frequency of ω and arbitrary direction of γ in the presence of arbitrary current with angle of β.

By substituting $F_i(\omega,\beta,\gamma)$ from Eq.(18), Eq.(19) and Eq.(20) into Eq.(21), the mean force components in irregular waves for surge, sway and yaw modes can be obtained as follows:

Surge drift force in irregular waves

$$\overline{F_1^s} = 2\int_0^\infty S(\omega)\frac{2}{3}C_T\left(1+\frac{2\omega U\cos\beta}{g}\right)\rho g r\cos\gamma d\omega$$

where $S(\omega)$ is the wave spectrum and can be defined according to 15[th] ITTC as:

$$\frac{S(\omega)}{H_s^2T_1} = \frac{0.11}{2\pi}\left(\frac{\omega T_1}{2\pi}\right)^{-5}\exp\left[-0.44\left(\frac{\omega T_1}{2\pi}\right)^{-4}\right]$$

where:

$$\int_0^\infty S(\omega)d\omega = m_0$$

$$\int_0^\infty \omega S(\omega)d\omega = m_1$$

$$\frac{m_1}{m_0} = \frac{2\pi}{T_1}$$

$$H_s = 4\sqrt{m_0}$$

Therefore, by substituting above equations in the main surge drift force equation in irregular waves we can now write:

$$\overline{F_1^s} = \rho g\frac{H_s^2}{16}C_T\left[1+F_{n1}\frac{4\pi}{T_1}\sqrt{\frac{L}{g}}\right]\left(\frac{4}{3}r\cos\gamma\right) \quad (22)$$

Sway drift force in irregular waves

Similarly to the surge equation, the following equation can be written to calculate the sway drift force in irregular waves.

$$\overline{F_2^s} = \rho g\frac{H_s^2}{16}C_T\left[1+F_{n2}\frac{4\pi}{T_1}\sqrt{\frac{B}{g}}\right]\left(\frac{4}{3}r\sin\gamma+L_2\sin\gamma|\sin\gamma|\right) \quad (23)$$

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Yaw drift force in irregular waves:

$$\overline{F_6^s} = \rho g \frac{H_s^2}{16} C_T \left[1 + F_{n2} \frac{4\pi}{T_1}\sqrt{\frac{B}{g}}\right] \begin{pmatrix} \frac{2}{3} r \sin\gamma \left(L_1' - L_2'\right) + \\ \left(L_2'^2 - L_1'^2\right)\sin\gamma|\sin\gamma| + \\ \frac{1}{3} r \left(L_1' + L_2'\right)\sin 2\gamma \end{pmatrix} \quad (24)$$

3.4.STEADY DRIFT MOTIONS

The steady drift motions for a moored floating structure can be obtained as follows:

$$\eta_1^s = \frac{F_1^s}{K_x} \quad (25)$$

$$\eta_2^s = \frac{F_2^s}{K_y} \quad (26)$$

$$\eta_6^s = \frac{F_6^s}{K_\theta} \quad (27)$$

where:

η_1^s, η_2^s and η_6^s are steady drift displacement in surge, sway and yaw modes respectively.

F_1^s, F_2^s and F_6^s are steady drift loads in irregular waves in surge, sway and yaw modes respectively.

K_x and K_y are mooring stiffness coefficients in x and y directions respectively.

K_θ is the rotational mooring stiffness which for a turret moored FPSO can be defined as:

$K_\theta = L_m{}^2 K_x$

where

L_m is the distance between the turret mooring point and FPSO's centre of gravity.

3.5.WAVE DRIFT DAMPING

Wave drift damping is one of the important parameters used for obtaining the response spectrum of moored floating offshore structures due to slow varying motions. Considering Eq.(14), the wave drift damping can be written as follows:

$$C_{ii} = C_T \omega \rho \zeta_a^2 \int_{L_1} \sin^2(\theta + \gamma) n_i dl \quad (28)$$

Therefore, wave drift damping in surge, sway and yaw will be:

$$C_{11} = \frac{4}{3} C_T \omega \rho \zeta_a^2 r \cos\beta \quad (29)$$

$$C_{22} = C_T \omega \rho \zeta_a^2 \left(\frac{4}{3} r \sin\gamma + L_2 \sin\gamma|\sin\gamma|\right) \quad (30)$$

$$C_{66} = C_T \omega \rho \zeta_a^2 \begin{bmatrix} \frac{1}{3} r \sin\gamma \left(L_1' - L_2'\right) + \\ \left(L_2'^2 - L_1'^2\right)\sin\gamma|\sin\gamma| + \\ \frac{1}{6} r \left(L_1' + L_2'\right)\sin 2\gamma \end{bmatrix} \quad (31)$$

The proposed wave drift damping in surge has been compared with the asymptotic formula presented by Faltinsen, Dahle et al. [15] and the results can be seen in Fig. 7. The shows a comparison between modified wave drift damping and asymptotic formula. It clearly shows that how wave drift damping coefficients in surge mode are non-linear in lower band of circular wave frequencies. It should be noted that the asymptotic formula is just valid for small wave lengths or in other words for high wave frequencies. But in the proposed formula as the finite draft coefficient has been considered in the mean wave drift force formula therefore, in the wave drift damping this coefficient appeared and it can be concluded that for low wave frequencies the proposed wave damping term could be valid as well as for high wave frequencies.

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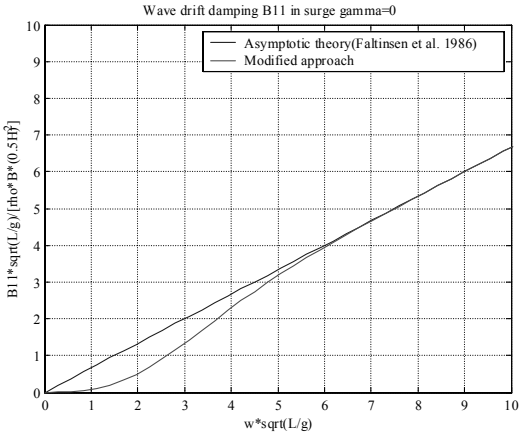


Fig. 7-Comparison between modified wave drift damping and asymptotic formul

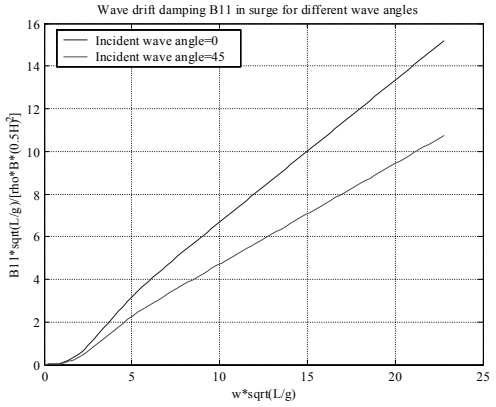


Fig. 8-The effect of wave heading angles on the wave drift damping (surge)

Also, the effect of wave heading angles on the wave drift damping in surge, sway and yaw can be seen in Fig. 8 to Fig. 10.

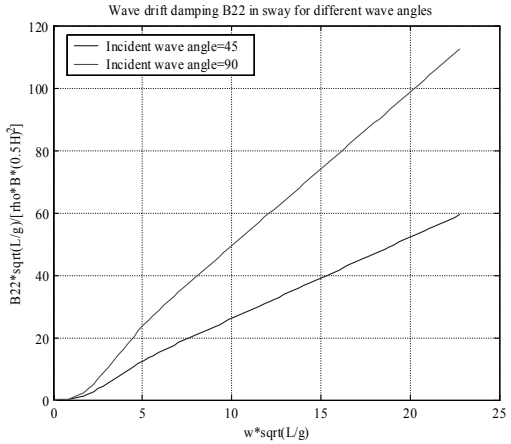


Fig. 9-The effect of wave heading angles on the wave drift damping (sway)

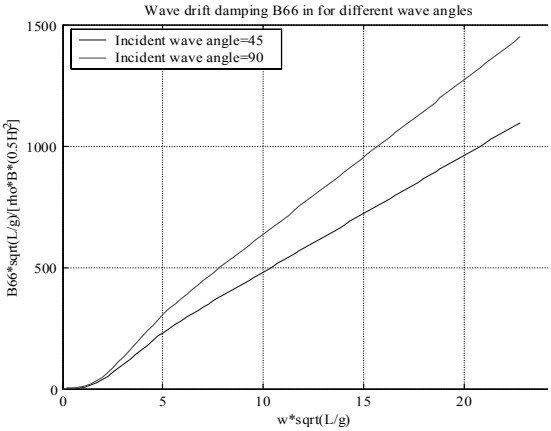


Fig. 10 -The effect of wave heading angles on the wave drift damping (yaw)

4. CONCLUDING REMARKS

The wave direction, wave frequency and waterline shape of a floating structures were considered to obtain the second-order mean wave loads on moored floating. In this regards, modification was made to Faltinsen's wave drift formula by adding finite draft coefficient. The modified formula can take into account the effects of wave direction, frequency and the waterline shape of floating structures. Meanwhile, the obtained formula was applied to an FPSO.. The results obtained from this formula which is dependent on wave frequencies, were compared with Helvacioglu's experiments [2] favorably in high wave frequencies. Moreover, the influence of the current on the wave mean drift force was taken into account by considering the current coefficient derived from the ship added resistance formula. It is found that, the presence of current can increase the mean drift force by up to 50 percent for a particular range of wave frequencies and, the amount of increase for a floating structure is independent of its size.

For a 200,000 tdw FPSO, the mean drift loads in irregular waves with H_s=5 m and T_1=16 sec are 5-15 percent of the mean drift loads of the mentioned FPSO when subjected to a regular wave with

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H =5 m and T =16 sec (Table 1, Table 2 and Table 3).

The formula for the calculation of the wave drift damping was extended to cover high wave frequencies as well as low wave frequencies. The results compared with asymptotic formula showed good agreement in the high frequency band.

Table 1-Mean drift load components on a case study FPSO ($\gamma = 0$)

Current Condition / Sea condition		U =0 m/s β =0 F_{n1} =0 F_{N2} =0	U =0 m/s β =0 F_{n1} =0.02 F_{N2} =0	U =1 m/s β =45 F_{n1} =0.013 F_{N2} =0.033	U =1 m/s β =90 F_{n1} =0 F_{N2} =0.047
Regular waves H =5 m T =16 sec	Surge (KN)	988.8	1067	1044	988.8
	Sway (KN)	0	0	0	0
	Yaw (KN.m)	0	0	0	0
Irregular waves H_s =5 m T_1 =16 sec	Surge (KN)	123.6	133.42	130.54	123.6
	Sway (KN)	0	0	0	0
	Yaw (KN.m)	0	0	0	0

Table 2-Mean drift load components on a case study FPSO ($\gamma = 45$)

Current Condition / Sea condition		U =0 m/s β =0 F_{n1} =0 F_{N2} =0	U =0 m/s β =0 F_{n1} =0.02 F_{N2} =0	U =1 m/s β =45 F_{n1} =0.013 F_{N2} =0.033	U =1 m/s β =90 F_{n1} =0 F_{N2} =0.047
Regular waves H =5 m T =16 sec	Surge (KN)	699.19	754.71	738.44	699.16
	Sway (KN)	3865	3865	4082	4172
	Yaw (KN.m)	192340	192340	203150	207620
Irregular waves H_s =5 m T_1 =16 sec	Surge (KN)	87.4	94.34	92.3	87.4
	Sway (KN)	483	483	510.3	521.5
	Yaw (KN.m)	11683	11683	12339	12611

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Table 3-Mean drift load components on the case study FPSO ($\gamma = 90$)

Current Condition / Sea condition		U =0 m/s β =0 F_{n1} =0 F_{N2} =0	U =0 m/s β =0 F_{n1} =0.02 F_{N2} =0	U =1 m/s β =45 F_{n1} =0.013 F_{N2} =0.033	U =1 m/s β =90 F_{n1} =0 F_{N2} =0.047
Regular waves H =5 m T =16 sec	Surge (KN)	0	0	0	0
	Sway (KN)	7320.4	7320.4	7731	7902
	Yaw (KN.m)	188620	188620	199220	203600
Irregular waves H_s =5 m T_1 =16 sec	Surge (KN)	0	0	0	0
	Sway (KN)	915.05	915.05	966.4	987
	Yaw (KN.m)	11310	11310	11945	12208

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