Numerical Calculations of Ship Induced Waves

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Abstract

Nowadays, various numerical methods are developed to extend computational fluid dynamics in engineering applications. One of the most useful methods in free surface modeling is Boundary Element Method (BEM). BEM is used to model inviscid fluid flow such as flow around ships. BEM solutions employ surface mesh at all of the boundaries. In order to model the linear free surface, BEM can be modified to a simple form called Green Function Method (GFM).

In this paper a developed code is applied GFM to several problems including flows around ships and the results are presented.

To verify the accuracy and application of the code, it is applied to flows around an analytically body which is Wigley hull and results are in good agreement with available data.

The wave-making properties are also investigated. Variations of the wave pattern versus the speed are studied, and a quantitative investigation of the forces is performed. In addition, the effects of linear solution are studied. Presented results and computer code can be used to optimize the hull forms for decreasing wave making resistance.

Keywords: Free Surface, Wave Making resistance, Boundary Element Method, Green Function Method

Nomenclature:

Ω	: Three dimensional domain	n	: Direction of normal vector
Γ	: Boundary of domain	(x,y,z)	: Cartesian coordinates
ϕ	: Potential of velocity	(ξ,η,ζ)	: Coordinates of singular point
w	: Weight function	θ	: Direction on free surface
δ	: Dirac-Delta function	L	: Overall length of ship
G	: Green function	B	: Breadth of midship
K	: Wave number	T	: Draft
p	: Arbitrary point inside the domain	Fr	: Froude Number
α	: Internal angle at point p		

Introduction

Free surface waves created by ships cause one of the most important drag components which is called wave making resistance. Several methods are used to compute wave resistance. Due to the complicated hull forms and intricate boundary conditions, analytical methods are not applicable. In addition, obtaining the experimental results is very expensive. Therefore numerical methods are very attractive and improved in recent years.

Modern computer technology enables simulations of free surface flows around a realistic ship hull and greatly improves the accuracy of the predictions of linear ship motions. With the schemes of finite difference, finite element, and boundary element as the choice for the numerical algorithm, the boundary element method has been established as a popular approach for free surface wave computations owing to its efficiency, accuracy and flexibility. Potential flow based panel method are based on Green's theorem which relates properties of flows within a domain to the domain boundary conditions. The pioneering work of Hess and Smith [1] change panel methods into the numerical calculation and simulation of potential flows for bodies of general shapes. There are normally two types of approaches towards the numerical solution of free surface flows. The first one is to adopt linearized free surface wave Green Function as the singularities distributed on the submerged hull surface and the uniform as the basis flow (Liapis [2], King [3], Beck and Magee [4], Korsmeyer [5] and Bingham [6]). While this method is elegant in enforcing free surface conditions and radiation conditions (the linearized free surface conditions are satisfied automatically and there is no need to discretize the free surface domain), it is difficult to extend the scheme to nonlinear solutions (Sclavounos [7]). The other approach, the Rankine Panel Method, was first introduced by Gadd [8] and Dawson [9], who employed the double body flow as the basis for linearization, chosen primarily trough physical intuition. The free surface is discretized into quadrilateral panels and covered by the Rankine sources and dipoles. This so called Rankine Panel Method (RPM) provides much flexibility for different kinds of free surface formulations and numerical algorithms, and models both steady and unsteady wave flows (Nakos [10], Raven [11], Jensen, Bertram and soding [12] and Kring [13]).

In this paper, a kind of GFM for steady-state free-surface potential-flow problem is employed to develop a computer code which calculates wave making resistance and the wave pattern of ships. This method uses a distribution of sources located on the wetted surface of the ship to represent the velocity potential.

Governing Equations

We can develop the boundary element method for the solution of Laplace's Equation ($\nabla^2\phi = 0$) in a three dimensional domain Ω. The basic steps are in fact quite similar to the finite element method .We firstly must form an integral equation from the Laplace's Equation by using a weighted integral equation and then use the Green-Gauss theorem. By using weighted integral equation is resulted:

$$0=\int_{\Omega}\nabla^{2}\phi.wd\Omega=\int_{\Gamma}\frac{\partial\phi}{\partial n}wd\Gamma-\int_{\Omega}\nabla\phi.\nabla w.d\Omega \tag{1}$$

Equation (1) is the starting point for the finite element method. To derive the starting equation for the boundary element method we use the Green-Gauss theorem again on the second integral. This gives:

$$\begin{aligned}0&=\int_{\Gamma}\frac{\partial\phi}{\partial n}wd\Gamma-\int_{\Omega}\nabla\phi.\nabla wd\Omega\\&=\int_{\Gamma}\frac{\partial\phi}{\partial n}wd\Gamma-\int_{\Gamma}\phi\frac{\partial w}{\partial n}d\Gamma+\int_{\Omega}\phi\nabla^{2}wd\Omega\end{aligned} \tag{2}$$

For the boundary element method we choose w to be the fundamental solution of Laplace's Equation. The fundamental solution for Laplace's Equation is a solution of:

$$\nabla^{2}w+\delta(\xi-x,\eta-y,\zeta-z)=0 \tag{3}$$

We solve the above equation without reference to the original domain G or original boundary conditions. In other words, the method is to try and find solution to $\nabla^{2}w=0$ in 3D which contains a singularity at the point (ξ,η,ζ). If w satisfies the linearized free surface conditions is no need to discretize the free surface domain. Wehausen & Laitone [14] obtained the solution for linear free surface around ships. They introduced w for deep water as:

$$\begin{aligned}&w=\frac{1}{2\pi}G\\&\nabla^{2}G=0\\&1)G_{\xi\xi}(x,y,z,\xi,\eta,0)+KG_{\zeta}=0\\&2)Lim_{Z\to-\infty}G_{\zeta}=0\end{aligned} \tag{4}$$

Green function is solution of equation (4) with boundary conditions 1 and 2, which are related to free surface and sea bottom conditions. Also one other condition is needed for ship downstream wave radiation.

$$\begin{aligned}&G(x,y,z,\xi,\eta,\zeta)=\frac{1}{r}-\frac{1}{r_{1}}\\&-\frac{4}{\pi}K\int_{0}^{\pi/2}d\theta\sec^{2}\theta\int_{0}^{\infty}dk\frac{\exp k(y+\eta)}{k-K\sec^{2}\theta}\cos[k(x-\xi)\cos\theta]\cos[k(y-\eta)\sin\theta]\\&-4K\int_{0}^{\pi/2}d\theta\sec^{2}\theta\exp[K(z+\zeta)\sec^{2}\theta]\sin[K(x-\xi)\sec\theta]\cos[K(y-\eta)\sin\theta\sec^{2}\theta]\\&r=\left[(x-\xi)^{2}+(y-\eta)^{2}+(z+\zeta)^{2}\right]^{1/2}\\&r_{1}=\left[(x-\xi)^{2}+(y-\eta)^{2}+(z-\zeta)^{2}\right]^{1/2}\end{aligned} \tag{5}$$

Thus we get the boundary integral equation:

$$c(p)\phi(p)+\int_{\Gamma}\phi\frac{\partial w}{\partial n}d\Gamma=\int_{\Gamma}\frac{\partial \phi}{\partial n}wd\Gamma$$

$$c(p)=\begin{cases}1 & p\in\Omega \\ \frac{1}{2} & p\in\Gamma \quad \text{Smooth Point on the Boundary} \\ \frac{\alpha}{4\pi} & p\in\Gamma \quad \text{Non Smooth Point on the Boundary}\end{cases} \qquad (6)$$

If point p happens to lie at some non smooth point e.g. a corner, then the coefficient $c(p)$ is replaced by $\frac{\alpha}{4\pi}$ where α is the internal angle at point p.

Equation (6) involves only the surface distributions of ϕ and $\frac{\partial \phi}{\partial n}$ and the value of ϕ at a

point p. Once the surface distributions of ϕ and $\frac{\partial \phi}{\partial n}$ are known, the value of ϕ at any point p inside Ω can be found since all surface integrals in Equation (6) are then known. The procedure is thus to use Equation (6) to find the surface distributions of ϕ and $\frac{\partial \phi}{\partial n}$ and then (if required) use Equation (6) to find the solution at any point $p\in\Omega$. Thus we solve for the boundary data first, and find the volume data as a separate step.

Since Equation (6) only involves surface integrals, as opposed to volume integrals in a finite element formulation, the overall size of the problem has been reduced by one dimension (from volumes to surfaces). This can result in huge savings for problems with large volume to surface ratios (*i.e.,* problems with large domains). Also the effort required to produce a volume mesh of a complex three-dimensional object is far greater than that required to produce a mesh of the surface.

Thus the boundary element method offers some distinct advantages over the finite element method in certain situations.

Geometry and Grid

The ship model employed in this study is a analytical Wigley hull form. The Wigley parabolic hull is characterized by sharp edges at the bow, stern and keel. It is mathematical form defined by:

$$|y|=\frac{B}{2}(1-\frac{4x^2}{L^2})(1-\frac{z^2}{T^2})$$

$$-\frac{L}{2}<x<\frac{L}{2} \qquad (7)$$

$$-T<z<0$$

$$0<y<\frac{B}{2}$$

In the (x,y,z) coordinates system with increasing values of x in the direction opposite to the ship's forward motion, z vertically upward, and the origin at the undisturbed level of the free surface. Here L is the length of the model, B is the beam at midship and T is the draft. For the selected form, the parametric values are $\frac{L}{B} = 10$ and $\frac{B}{T} = 1.6$ (Fig.1).

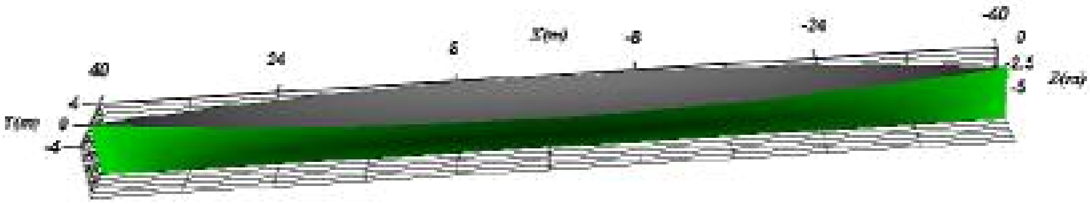

Figure 1- Wigley hull form

Numerical solutions use discretized geometry. In order to discretize boundaries (wetted surface of ship) in GFM, triangular elements are used. Triangular elements can fit on hull form perfectly. Figure (2) displays grid employed in this work. Due to symmetric condition of hull form, only half body is modeled.

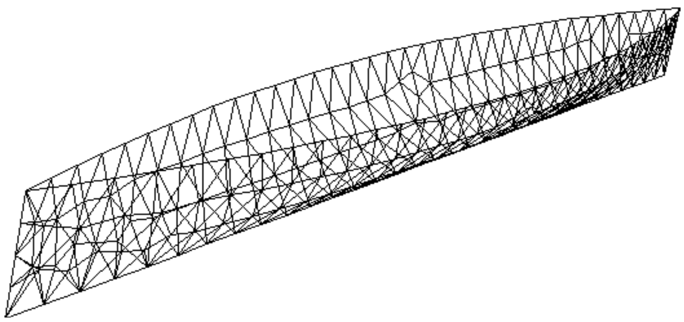

Figure 2- Triangular elements on hull form

Results

The solution algorithm outlined in the preceding sections has been applied to the prediction of wave making resistance coefficient at a short range of Froude numbers. The wave drag was computed by integrating the pressure over the wetted surface.

The results are obtained due to Froude number changing in range of 0.2 to 0.4. Similar other numerical methods, the first step for getting correct data is checking mesh independency. Figure (3) shows wave making resistance coefficient due to mesh size number on half body. By increasing mesh size over 200 elements, varying wave making resistance coefficient can be ignore.

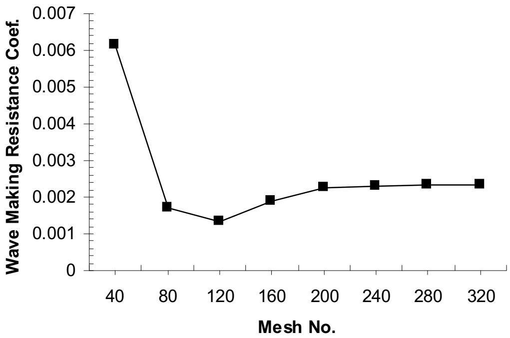

Figure 3 - Mesh Independency

Four Froude Numbers are considered for modeling. Figure (4) shows a comparison of computed wave profile for Wigley hull form with experimental data at Fr=0.25. The wave length of computed and experimental wave profiles is in fairly good agreement. The linearization free surface condition causes error at the stagnation point of fore ship.

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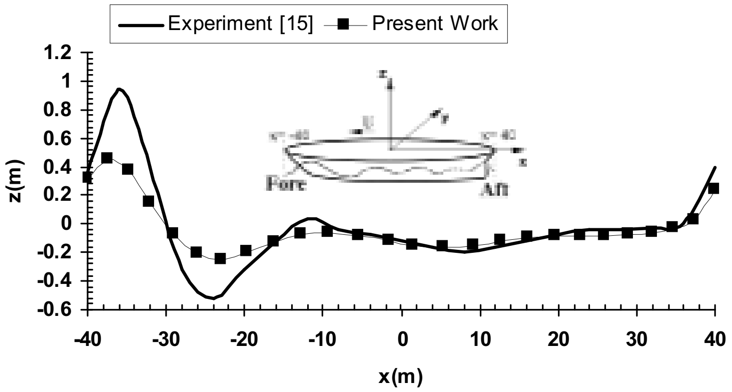

Figure 4 - Comparison of computed with experimental wave profile at Fr=0.25

Wave pattern at Fr=0.25 is illustrated in figure (5) and the Kelvin wave angles are measured. There is a good agreement between the numerical and theoretical wave angles.

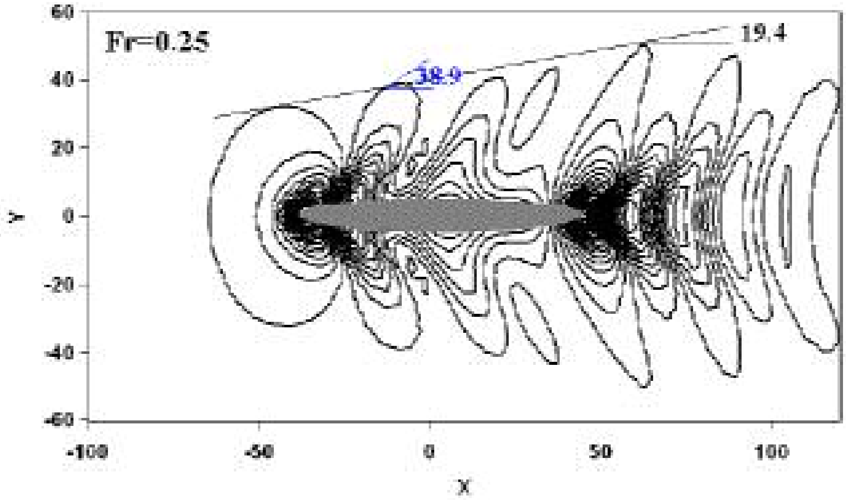

Figure 5 - Wave pattern at Fr=0.25

Figure (6) shows contours of pressure coefficient and components of velocity on the free surface at Fr=0.4.

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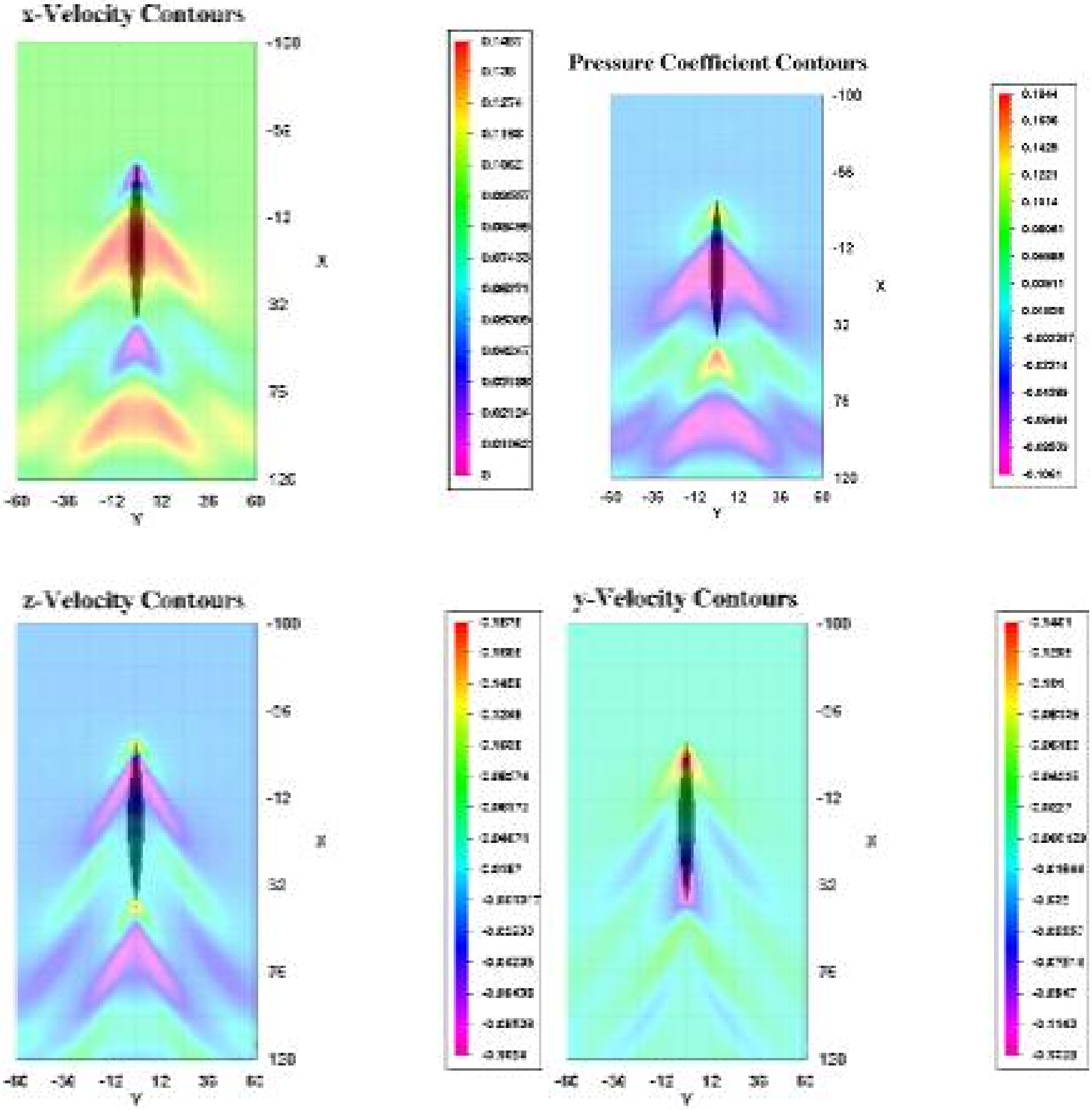

Figure 6 - Contours of pressure coefficient and component of velocity

The calculated free surface elevations along the centerline of the domain are shown in Figures (7), showing the effect of a variation in Froude number. The bow (or front) of the hull is located at x=-40 and the stern (or rear) is located at x=40. As can be seen, the Froude number has a strong effect on the wave profile.

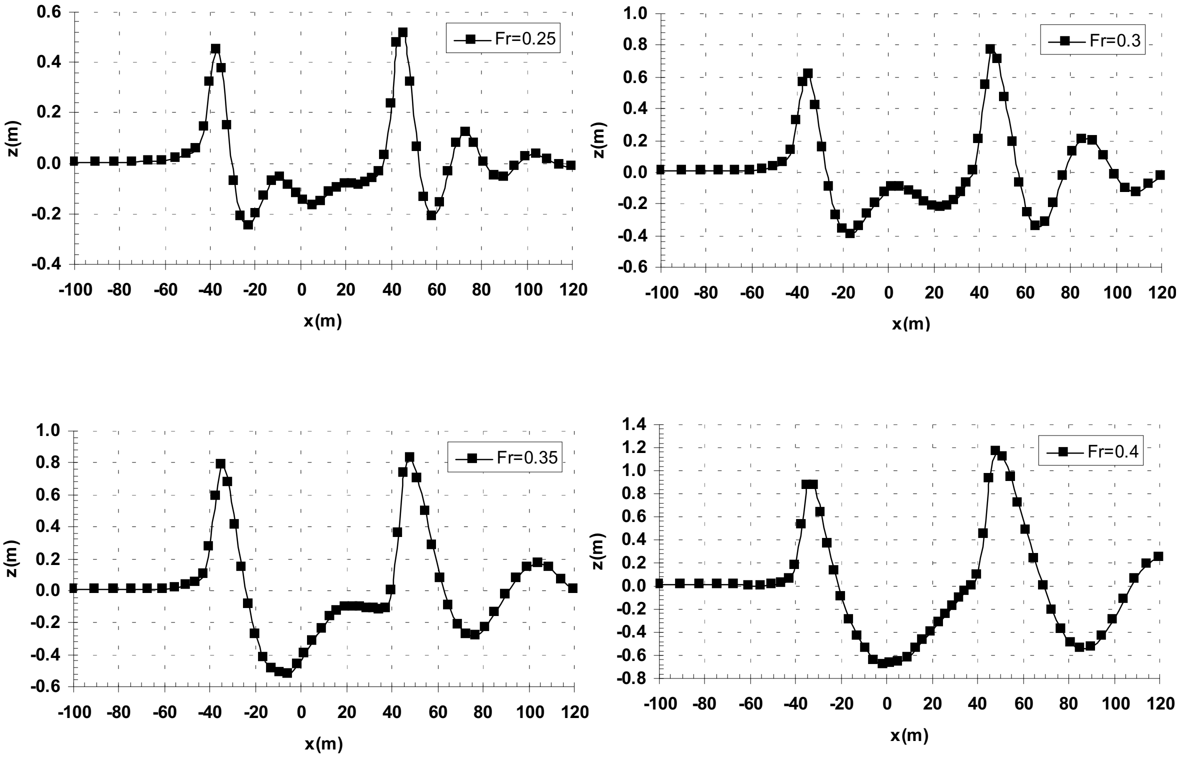

Figure 7- Wave profile on centerline

Figure (8) shows a comparison of the computed wave drag coefficient with experimental data for the Wigley hull model. Numerical predictions are in fair agreement with experimental data.

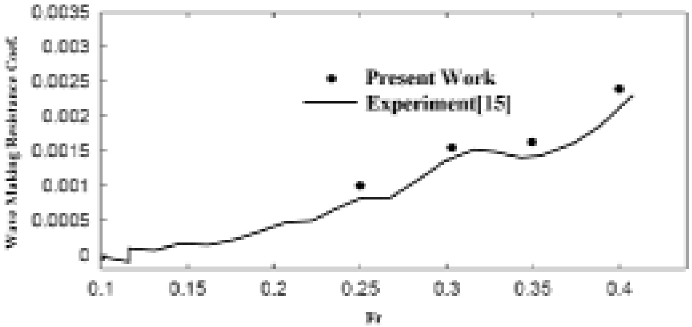

Figure 8 - Comparison of Wave making resistance coefficient

Figure (9) shows the wave patters for the Wigley hull model. The wave patterns are plotted using the same contour level for four Froude numbers considered. These two figures show the change of wave patterns with Froude numbers.

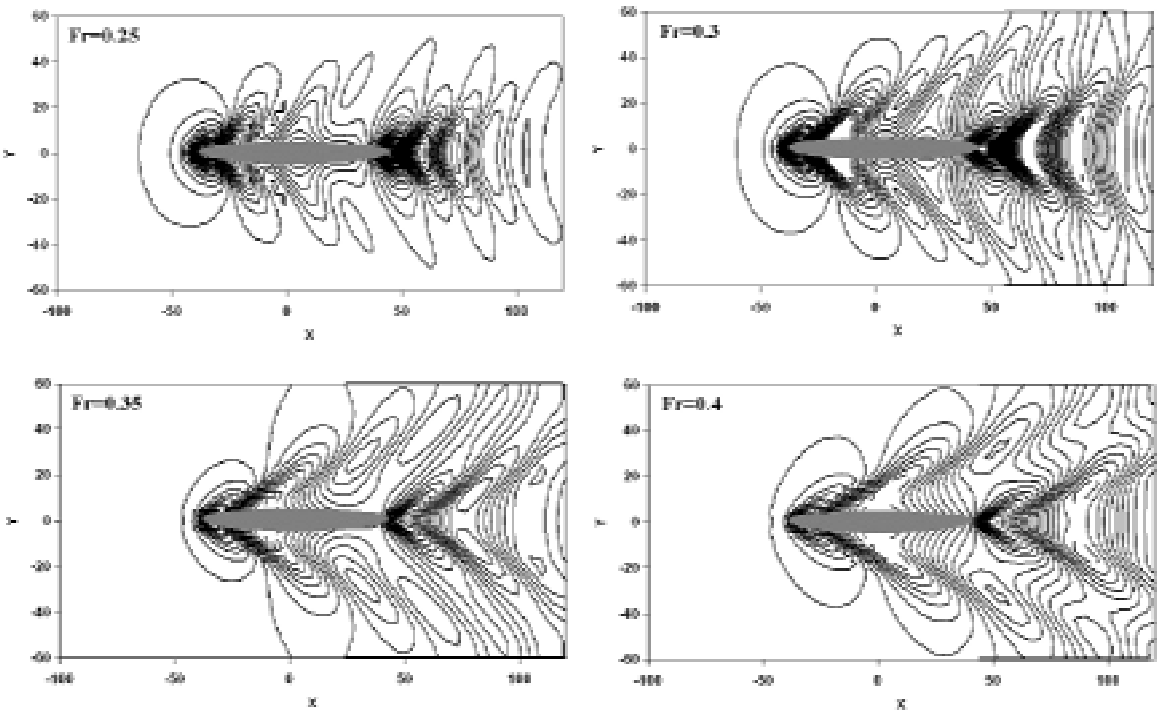

Figure 9 - Wave patterns

Conclusion

A Green Function Method has been extended to account for steady ship waves. The wave profiles and wave drag computed using the present approach are in good agreement with experimental measurements for a mathematical hull form at a range of Froude numbers. Also, the wave angles are as those predicted by Kelvin's ship wave theory.

Presented results and computer code can be used to optimize the hull forms for decreasing wave making resistance.

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